UNIVERSITY OF CALIFORNIA BERKELEY
Department of EECS

Problem 1: Got Filters?

Your friend Arif, who dabbles in stock and foreign-exchange trading on the international financial markets, wants to tell you about a simplified version of one of the tools they use in the "technical analysis" of price data.

Let $x[n]$ be the closing price of stock X on day n. Arif tells you about filters that they use to determine price trends. In particular, he introduces you to a filter that takes the stock price data signal x and produces another signal y according to the following equation:

$$\forall n \in \mathbb{Z}, \quad y[n] = \frac{1}{3}\big[x[n] + x[n-1] + x[n-2]\big].$$

(a) Explain, in plain English, what Arif's toy filter does to the input stock-price signal x.

(b) Determine the impulse response of the filter, and provide a well-labeled plot of it. That is, determine and plot the response of the filter corresponding to the input signal given by

$$x[n] = \delta[n] \triangleq \begin{cases} 1 & n = 0 \\ 0 & n \neq 0. \end{cases}$$

(c) Determine, and provide well-labeled plots of, the filter's responses y_1, y_2, and y_3 corresponding to the following hypothetical signals (which do not necessarily represent stock price data):

$$x_1[n] = 1 \text{ for all } n \in \mathbb{Z}$$

$$x_2[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases} \quad \text{(Unit-Step Signal)}$$

$$x_3[n] = \begin{cases} 1 & \text{if } n \text{ is even} \\ -1 & \text{if } n \text{ is odd.} \end{cases}$$

Based on what you discovered about y_2, explain why Arif would call the filter's output a *lagging indicator* of the stock price's movement.

Problem 2:

This problem explores some consequences of Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

(a) $\cos\,\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$

(b) $\sin\,\theta = \frac{e^{i\theta}-e^{-i\theta}}{2i}$

(c) Derive De Moivre's Theorem, where for all integers n,

$$(\cos\theta + i\,\sin\theta)^n = \cos(n\,\theta) + i\,\sin(n\,\theta).$$

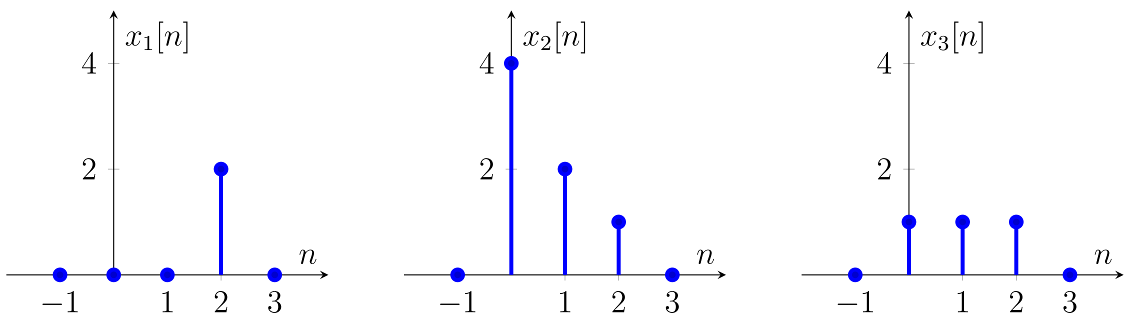
(d) Using Euler's Formula with a strategically chosen value of θ, derive what American mathematician Benjamin Peirce referred to as the "mysterious formula" [1],

$$\pi = \frac{2}{i}\ln(i).$$

[1] For more on this, read the wonderful book by Paul J. Nahin, *An Imaginary Tale: The Story of* $\sqrt{-1}$, Princeton University Press, 1998, ISBN: 0-691-12798-0

Problem 3: DT Convolution (Practice)

Consider the three discrete-time signals below.



Carefully sketch and label plots of the following signals.

(a) $(x_1 * x_2)[n]$

(b) $(x_3 * x_3)[n]$

Problem 4: Is it LTI?

Each of the following equations describes a relationship that holds between an input signal x and an output signal y of an associated discrete-time system, for all integers n. In each case, determine if the system is (i) linear and (ii) time-invariant.

(a) $y[n] = ax[n - n_0] + b$, where $a, b \in \mathbb{C}, n_0 \in \mathbb{Z}$. In cases where you answer no: does your answer become yes change if we fix $b = 0$? What if we fix $n_0 = 0$?

(b) $y[n] = x[n] - x[n - 1]$

(c) $y[n] = \ln(x[n])$

(d) $y[n] = x[2n]$