Problem 1: DTFT Practice

An LTI system with impulse response $h_1[n] = (1/3)^n u[n]$ is connected in *parallel* with another causal LTI system with impulse response $h_2[n]$. The resulting parallel interconnection has the frequency response

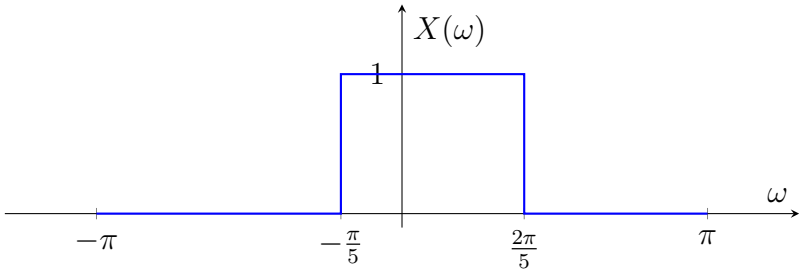
$$\begin{aligned} H(\omega) &= \frac{-12 + 5e^{-i\omega}}{12 - 7e^{-i\omega} + e^{-i2\omega}} \\ &= \frac{-3}{e^{-i\omega} - 3} + \frac{8}{e^{-i\omega} - 4} \end{aligned}$$

(a) Find $h_2[n]$ using a DTFT pair derived in class:

$$\alpha^n u[n] \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{1 - \alpha e^{-i\omega}} \quad |\alpha| < 1$$

For the rest of this problem, we will use Fourier transform properties to either compute the DTFT or inverse DTFT of signals.

(b) The spectrum $X(\omega)$ is shown below for $\omega \in [-\pi, \pi]$, and 2π-periodically repeats outside this region. **You do not need to compute any integrals.**



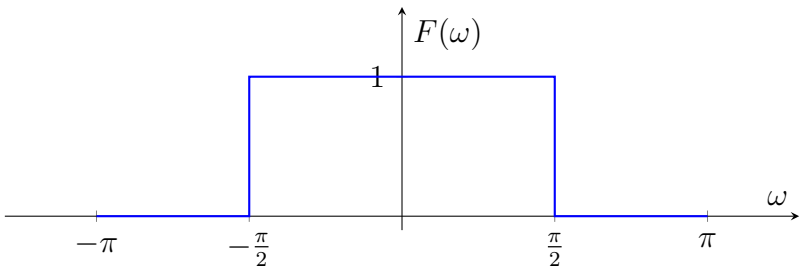
Feel free to use the fact that the inverse DTFT of a spectrum defined as

$$F(\omega) = \begin{cases} 1 & \omega \in [-\omega_c, \omega_c] \\ 0 & \text{otherwise} \end{cases}$$

for $|\omega| < \pi$ (repeating 2π-periodically outside that region) is

$$f[n] = \frac{\sin(\omega_c n)}{\pi n}.$$

(c) We have some system F whose impulse response $f[n]$'s DTFT $F(\omega)$ is shown below for $\omega \in [-\pi, \pi]$, and 2π-periodically repeats outside this region.:



If the input $x[n] = \frac{1}{2}\delta[n-1] + \frac{1}{2}\delta[n+1]$ is passed into this system, what is the corresponding DTFT $Y(\omega)$ of the output?

Problem 2: Mechanical CTFT

For each of the $x(t)$ below, find $X(\omega)$.

(a) $x(t) = e^{i\omega_0 t}$.

(b) $x(t) = \cos(\omega_0 t + \phi)$.

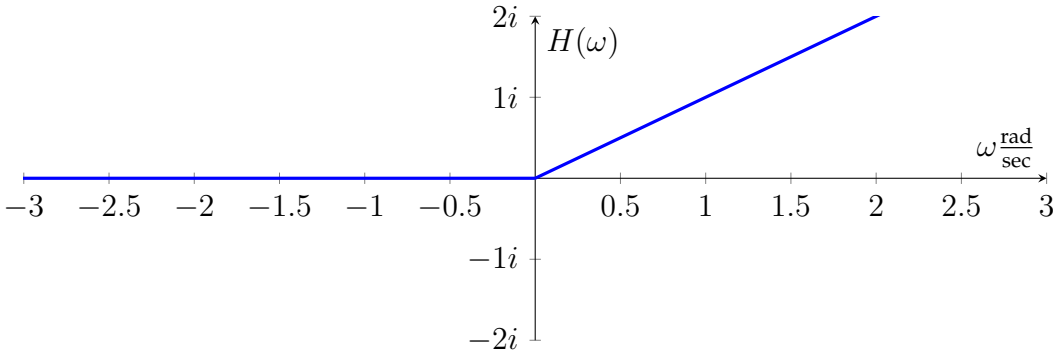
(c) $x(t) = Y(t)$ where $Y(\omega)$ is the Fourier transform of some signal $y(t)$.

(d) $x(t) = R_T(t)$ where $R_T(t)$ is a length T rect:

$$R_T(t) = \begin{cases} 1 & |t| < T \\ 0 & \text{else} \end{cases}$$

(e) $x(t) = \frac{\sin(Tt)}{\pi t}$.

Problem 3: Derivative



$H(\omega)$ is the frequency response of an LTI system and $x(t)$, $y(t)$ is an input-output pair for the system, where

$$x(t) = \begin{cases} e^{i\omega_0 t}\frac{\sin(\omega_0 t)}{\pi t} & t \neq 0 \\ \frac{\omega_0}{\pi} & t = 0 \end{cases}$$

Find $y(t)$ when $t \neq 0$.

Hint: Write $Y(\omega)$ in terms of $X(\omega)$.