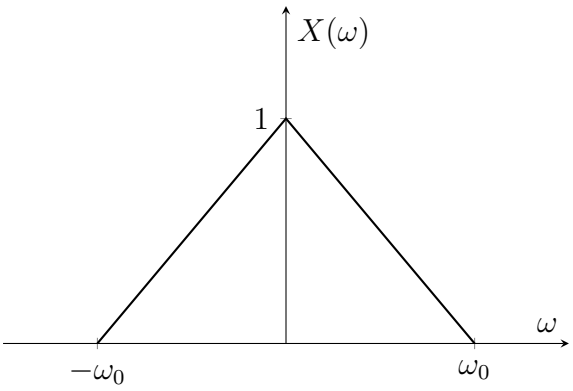
Problem 1: Aliasing

A real analog signal $x(t)$ has CTFT $X(\omega)$:



We define the impulse train

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

for sampling rate T.

(a) Let $\omega_N = \frac{2\pi}{T_N}$ be the Nyquist frequency corresponding to a signal with the bandwidth of $x(t)$ above. What is T_N? Sketch $P(\omega)$, the CTFT of $p(t)$, when $T = T_N$.

(b) Sketch $X_N(\omega)$, the CTFT of $x_N(t) = x(t)p(t)$, when $T = T_N$.

(c) Let $x_A(t)$ be $x(t)$ sampled at frequency $\omega_s = 2\omega_N$. Sketch $X_A(\omega)$.

(d) Let $x_B(t)$ be $x(t)$ sampled at frequency $\omega_s = \frac{3\omega_0}{2}$. Sketch $X_B(\omega)$. Indicate the region where aliasing occurs.

(e) Sketch $H(\omega)$, the the CTFT of $h(t)$, the ideal reconstruction filter for a signal with the same bandwidth as $x(t)$.

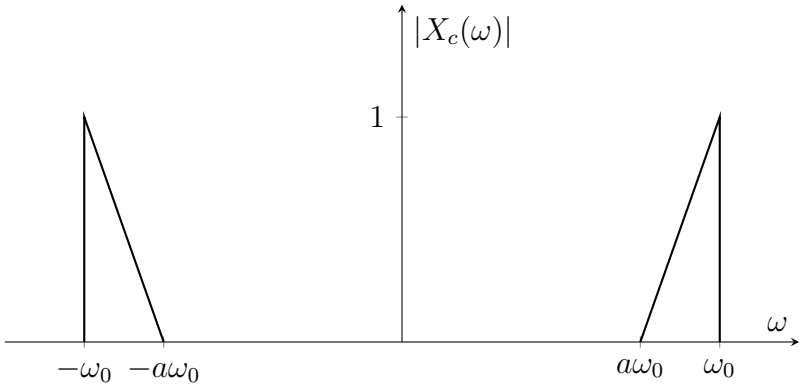
(f) Draw $X_{BR}(\omega)$, the CTFT of $x_{BR}(t) = x_B(t) * h(t)$.

(g) Suppose that instead of the spectrum given above, we had $x(t) = \cos(\omega_0 t)$. Now what is $X_{BR}(\omega)$?

(h) For $x(t) = \cos(\omega_0 t)$, draw $x(t)$ and $x_{BR}(t)$ on the same plot. Circle the points where $x(t)$ was sampled to generate $x_B(t)$.

Problem 2: Sampling Below Nyquist

An analog signal $x_c(t)$ has CTFT $X_c(\omega)$ with magnitude as shown in the following figure (where $0 < a < 1$):



(a) Sketch $|X_p(\omega)|$, the magnitude of the CTFT of $x_p(t) = x_c(t)p(t)$ where

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

for $T = \frac{2\pi}{\omega_0}$. Assume $a = \frac{3}{4}$.

(b) Devise a scheme that could recover $x_c(t)$ from $x_p(t)$. Illustrate or draw a block diagram.

(c) For what range of values of a, $0 < a < 1$, could $x_c(t)$ be recovered from $x_d(t)$ sampled with $T = \frac{2\pi}{\omega_0}$?

(d) With $a = \frac{3}{4}$, what is the maximum value of T for which $x_c(t)$ could be recovered from $x_p(t)$? How does this compare to the Nyquist rate?

Problem 3: Interpolation Methods

Recall that given a uniformly sampled version of a continuous time signal, we can reconstruct the original signal by low-pass filtering (multiplying by a box) in the frequency domain. In time, this corresponds to convolution with a sinc, where each Dirac delta in the sampled signal has an associated sinc function. This is known as sinc interpolation.

Sinc interpolation would give a perfect reconstruction of the original signal, but there are many other methods of interpolation in signal processing, some of which we will be exploring in this problem.

(a) Zero-order hold is a method of interpolation where reconstruction consists of holding the output signal at the height of each sample until the next height is encountered. What signal does this correspond to convolution with in time?

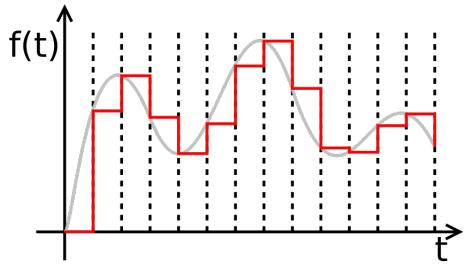


Figure 1: Zero-order hold example

(b) Qualitatively, what does the magnitude of the frequency response look like

when plotted? Why does reconstruction using zero-order hold work somewhat, but not as well as sinc interpolation?

(c) First-order hold is a method of interpolation where reconstruction consists of connecting the heights of each delta linearly in order to form the output signal. What signal does this correspond to convolution with in time?

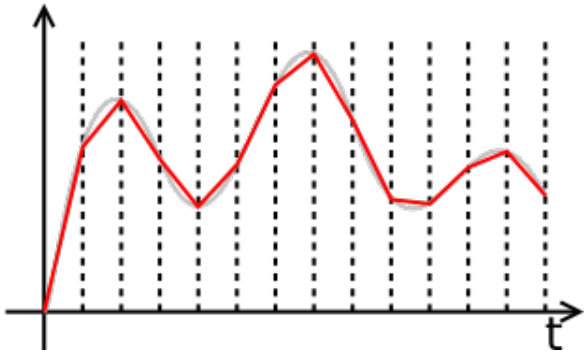


Figure 2: First-order hold example

(d) Qualitatively, what does the magnitude of the frequency response look like when plotted? Why does reconstruction using first-order hold work better than zero-order hold?