UNIVERSITY OF CALIFORNIA BERKELEY
Department of EECS

Problem 1: Understanding System Properties

Each of the following equations describes a relationship that holds between an input signal x and an output signal y of an associated discrete-time system, for all integers n. In each case, determine if the system is (i) memoryless, (ii) causal, (iii) BIBO stable.

(a) $y[n] = 7x[n+1]$

(b) $y[n] = ax[n-n_0] + b$, where $a, b \in \mathbb{C}, n_0 \in \mathbb{Z}$. In cases where you answer no: does your answer become yes change if we fix $b = 0$? What if we fix $n_0 = 0$?

(c) $y[n] = x[n]x[n-1]$

(d) $y[n] = e^{x[n]}$

(e) $y[n] = x(-n)$

(f) $y[n] = v[n]x[n]$, where v is a fixed, *bounded* discrete-time signal.

Problem 2: Dirac Delta and CT Convolution Practice

In discrete-time, we talked about the notion of an impulse response as a system's response to a Kronecker delta, $\delta(n)$. In continuous-time, we have a similar notion, but we must use the Dirac delta, $\delta(t)$. Similarly, continuous-time convolution is defined as

$$x * y(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, \mathrm{d}\tau \tag{1}$$

The Dirac delta has the following properties:

- $\delta(t)$ is zero $\forall t \neq 0$, and is infinite-valued at $t = 0$.
- $\int_A^B \delta(t)dt = 1$ if $A < 0 < B$ and 0 otherwise.
- **Sampling property**: $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$.
- **Sifting property**: $\int_A^B x(t)\delta(t-t_0)dt = x(t_0)$ if $A < t_0 < B$ and 0 otherwise.
- **Convolution property**: If $y(t) = \alpha\delta(t-t_0)$, then $(x * y)(t) = \alpha x(t-t_0)$.
- **Time-scaling property**: $\delta(at) = \frac{1}{|a|}\delta(t)$.

(a) Evaluate the following expressions.

(i) $\int_{-\infty}^{\infty} \left[3\delta(\tau) + 2\delta(\tau-1)\right] \mathrm{d}\tau$

(ii) $x(t) = \int_0^{\infty} \tau^2 \delta(t-\tau^2)\, \mathrm{d}\tau$

(iii) $u * u(t)$, where $u(t)$ is the continuous-time unit step function:

$$u(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases} \tag{2}$$

(b) Prove the convolution property of the Dirac delta, i.e., derive an expression for $x(t) * \alpha\delta(t - t_0),\ \alpha, t_0 \in \mathbb{R}$, in terms of $x(t)$, where $x(t)$ is some continuous-time signal.