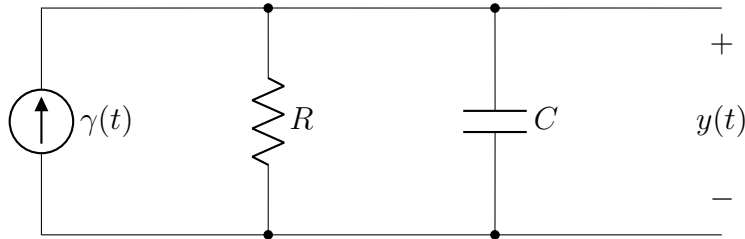


Problem 1: Parallel RC Circuit

Consider the following parallel RC circuit, driven by a current source $\gamma(t)$, with output $y(t)$ the voltage across the parallel combination of R and C .



By KCL at the top node, the source current splits between the capacitor ($C\dot{y}$) and the resistor (y/R), so the system is described by the differential equation

$$C\dot{y}(t) + \frac{1}{R}y(t) = \gamma(t)$$

Note: You don't need to know how to read the circuit to answer this problem.

(a) Find the impulse response $h(t)$. Assume the form of impulse response $h(t) = Ae^{st}u(t)$.

(b) Find the frequency response $H(\omega)$ using the eigenfunction property.

(c) Find the frequency response using the formula $H(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$.

(d) Calculate and sketch the magnitude response $|H(\omega)|$ and $\angle H(\omega)$ as a function of the frequency ω . Is it a low-pass, high-pass or band-pass filter?

Problem 2: Causality and BIBO Stability, the Sequel

For each of the following systems, determine whether or not they are causal and/or BIBO stable. Be careful, as they are not necessarily LTI.

(a) $y(t) = x(-e^{-\alpha t})$, $\alpha \in \{-1, 0, 1\}$.

(b) $y(t) = \int_{-5|t|}^t x(\tau) d\tau$.

Problem 3: (RLC Circuit)

Consider the series RLC -circuit shown in Figure 1, where x and y are the input and output, respectively.

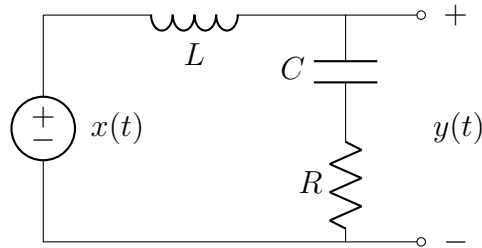


Figure 1: RLC circuit.

The transfer function $H(\omega)$, with the particular input and output signals specified in the figure above, is given by

$$H(\omega) = \frac{RC(i\omega) + 1}{LC(i\omega)^2 + RC(i\omega) + 1}. \quad (1)$$

Assume the system is initially at rest: $y(0^+) = \dot{y}(0^+) = 0$.

- (a) Determine the linear, constant-coefficient differential equation that governs the input-output behavior of the circuit.

(b) Using time domain analysis, derive the differential equation that governs the input-output behavior of the circuit. Does this match your answer from part a?

Hint: Here are some helpful equations for circuit analysis:

- For an inductor, $V_L = L \frac{dI_L}{dt}$.
- For a capacitor, $I_C = C \frac{dV_C}{dt}$.
- KCL: The sum of currents entering a node equals the sum of currents leaving the node.
- KVL: The sum of voltages in a closed loop is 0.

(c) Draw the integrator-adder-gain diagram for this system.