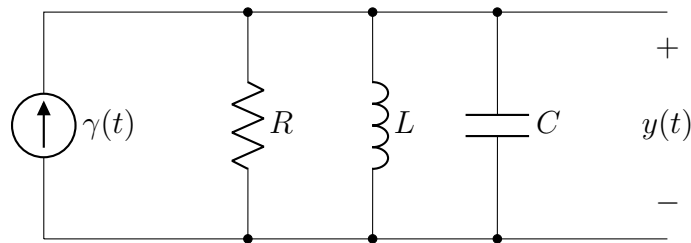


Problem 1: Parallel RLC Circuit

Consider the following parallel RLC circuit, driven by a current source $\gamma(t)$, with output $y(t)$ being the voltage across the parallel combination of R , L , and C . R , L , and C are all positive and real-valued.



By KCL at the top node, the source current splits between the capacitor ($i_C(t)$), the inductor ($i_L(t)$), and the resistor ($i_R(t)$):

$$\gamma(t) = i_R(t) + i_L(t) + i_C(t)$$

Note: You don't need to know how to read the circuit to answer this problem. The problem gives you all of the relevant equations.

The system dynamics are governed by Kirchhoff's Current Law (KCL) and the component constituent relations:

$$i_R(t) = \frac{y(t)}{R}, \quad i_C(t) = C\dot{y}(t), \quad y(t) = Li_L(t)$$

Define the following state variables:

1. $q_1(t) = i_L(t)$ (inductor current)
2. $q_2(t) = y(t)$ (output voltage)

Time-domain analysis of the circuit gives the following results:

$$\begin{aligned} \dot{i}_L(t) &= \frac{1}{L}y(t) \\ \dot{y}(t) &= -\frac{1}{C}i_L(t) - \frac{1}{RC}y(t) + \frac{1}{C}\gamma(t) \end{aligned}$$

- (a) Find the state-space matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ for the continuous-time state-space representation:

$$\dot{\mathbf{q}}(t) = \mathbf{A}\mathbf{q}(t) + \mathbf{B}\gamma(t)$$

$$y(t) = \mathbf{C}\mathbf{q}(t) + \mathbf{D}\gamma(t)$$

- (b) Consider the state matrix $\mathbf{A}$. Then, answer the following questions:

- (i) Is the system internally/asymptotically stable? Justify your answer.

(ii) Under what conditions (what values of R , L , C) is the system overdamped? Critically damped? Underdamped?

(iii) As R increases, how does the system's zero-input response change? Does the system become more heavily damped or less heavily damped?

(c) Find the frequency response $H(\omega)$ of the system using the eigenfunction property. Keeping in mind that $\dot{i}_L(t) = \frac{1}{L}y(t)$, start with the time-domain equation:

$$\gamma(t) = i_R(t) + i_L(t) + i_C(t) = \frac{1}{R}y(t) + i_L(t) + C\dot{y}(t)$$

(d) Find the magnitude $|H(\omega)|$ and phase $\angle H(\omega)$ of the frequency response. Sketch both plots, indicating key values on the axes, and state what type of filter this circuit represents.

(e) Draw an Integrator-Adder-Gain (IAG) block diagram representing this system. Clearly label the state variables $q_1(t)$ and $q_2(t)$.