

Problem 1: CTFS Practice

- (a) Find the Fourier series coefficients X_k for the following signal. Use the minimum period.

$$x(t) = \sin(\pi t) + 2 \cos(3\pi t)$$

- (b) Let $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{ik\omega_0 t}$ be a p -periodic signal.

- (i) Determine the Fourier coefficients of $y(t) = x(t - T)$, for some $T \in \mathbb{R}$.

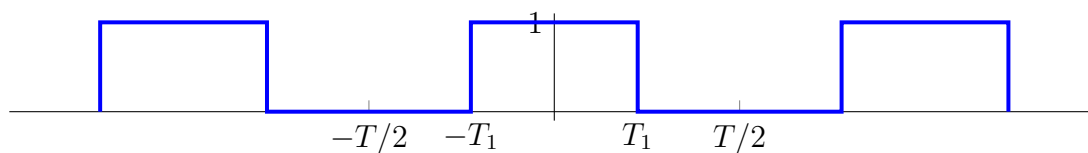
- (ii) Let $z(t) = e^{iM\omega_0 t} x(t)$, for some $M \in \mathbb{Z}$. Determine the Fourier coefficients of $z(t)$.

- (c) Given the Fourier series coefficients, determine the continuous time signal $x(t)$ with period $T = 4$:

$$X_k = \begin{cases} ik & |k| < 3 \\ 0 & \text{otherwise} \end{cases}$$

- (d) Find the Fourier coefficients X_k for the following signal. Assume $x(t)$ is periodic with period T .

$$x(t) = \begin{cases} 1 & |t| \leq T_1 \\ 0 & T_1 < |t| \leq \frac{T}{2} \end{cases}$$



Problem 2: DTFT Multiplication Property

A discrete-time signal and its Fourier transform are described below:

$$g(n) = \frac{B}{\pi n} \sin(An) \xleftrightarrow{\mathcal{F}} G(\omega) = \begin{cases} B & \text{if } |\omega| < A \\ 0 & \text{if } |\omega| \geq A \end{cases}$$

where $A, B \in \mathbb{R}$. It is understood $G(\omega)$ is 2π -periodic, and that $0 < A < \pi/2$.

Consider a related signal $h(n) = (g(n))^2$ for all $n \in \mathbb{Z}$.

(a) Plot and label $H(\omega)$, the DTFT of h .

Hint: recall that the multiplication property says

$$x(n) y(n) \xleftrightarrow{\mathcal{F}} \frac{1}{2\pi} (X \circledast Y)(\omega) = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X(\lambda) Y(\omega - \lambda) d\lambda$$

for any $x(n) \xleftrightarrow{\mathcal{F}} X(\omega)$ and $y(n) \xleftrightarrow{\mathcal{F}} Y(\omega)$.

(b) If H is the frequency response of a filter, and the input to the filter is

$$x(n) = 1 + \cos(An)$$

determine the corresponding output y .

Problem 3: Scaling, Revisited

Define the *area* of a discrete-time signal x as

$$A_x = \sum_{n=-\infty}^{\infty} x(n)$$

for all signals such that the sum converges. You may assume all signals in this question have finite area.

(a) Let $y(n) = (x * h)(n)$. Using the DTFT analysis equation, show that $A_y = A_x A_h$.

(b) Given a DT-LTI filter H with impulse response h , describe how to construct an equivalent¹ filter G with impulse response g that preserves the area of its inputs. That is, if y is the output of G to x , then it should hold that $A_y = A_x$.

¹For this problem, we say that two filters are equivalent if we can perform a simple operation, such as scaling by a constant or adding or subtracting a constant, to convert between their impulse responses.