Problem Set 2
Due: Fri, 4 Sep 2026, 23:59
Accepted Thru: Sun, 6 Sep 2026, 23:59

Your LAST Name ____________________ **Your FIRST Name** ____________________

Your Collaborators

- Name: ________________________________ SID: __________________________
- Name: ________________________________ SID: __________________________
- Name: ________________________________ SID: __________________________
- Name: ________________________________ SID: __________________________

Policy Statement

- We encourage you to collaborate, but only in a group of up to *five* current EE 120 students.
- Be sure to write the names of your collaborators at the appropriate location on the next page. Each collaborator must submit a distinct, self-prepared solution document containing original contributions to the collaborative effort.
- To submit your work, you have several options:
  (a) Print out this homework document and write your solutions on the printout. Then upload a *PDF file* of the scan.
  (b) Write your solutions on this PDF file using a tablet device. Then upload a *PDF file*.
  (c) Use your own sheets of paper, but demarcate clearly the space for each problem, part, and subpart. Then upload a *PDF file*.
- Please write neatly and legibly, because *if we can't read it, we can't evaluate it.* Remember that, even though the problem sets are self-graded, we will evaluate your work whenever we check your self grades.
- Unless we explicitly state otherwise, you may expect full credit *only if* you explain your work succinctly, but clearly and convincingly.
- If you are asked to provide a "sketch," it refers to a well-labeled *hand-drawn* sketch—not a plot generated by a computing device.
- On occasion, a problem set contains one or more problems designated as "optional." Your solutions to such problems will NOT be evaluated. Nevertheless, you're responsible for learning the subject matter within their scope.
- If you have a confirmed disability that precludes you from complying fully with these instructions or with any other parameter associated with this problem set, please alert us immediately about reasonable accommodations afforded to you by the DSP Office on campus.

Overview

This problem set explores some aspects of Euler's Formula; the convolution of discrete-time signals; the system properties of linearity and time-invariance; and the impulse responses and frequency responses of discrete-time LTI (DT-LTI) systems.

HW2.1 This problem explores another consequence of Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

Show that any linear combination of a set of cosine waves of frequency ω is always a cosine wave of the same frequency, *even if* each cosine wave has a distinct phase. In particular, show that

$$\sum_{k=1}^{N} A_k \cos(\omega t + \phi_k) = A\cos(\omega t + \phi).$$

Hint: Recognize that any complex number can be written as $Ae^{i\phi}$ for suitable A and ϕ. Further notice that $\sum_{k=1}^{N} A_k e^{i\phi_k}$ is a complex number.

HW2.2 (Is it LTI?) Each of the following equations describes a relationship that holds between an input signal x and an output signal y of an associated discrete-time system, for all integers n. In each case, determine if the system is (i) linear and (ii) time-invariant.

(a) $y[n] = 7x[n+1]$.

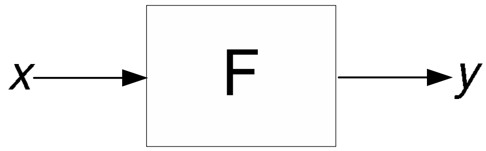
(b) $y[n] = x[n]x[n-1]$

(c) $y[n] = e^{x[n]}$

(d) $y[n] = x[-n]$

(e) $y[n] = v[n]x[n]$, where v is a fixed, non-constant discrete-time signal.

HW2.3 The impulse response of a discrete-time LTI system F



is the *discrete-time doublet*: $f[n] = \frac{1}{2}[\delta[n] - \delta[n-1]]$.

(a) For each of the inputs below, determine the corresponding output of the system. Be sure to provide a well-labeled sketch of all the signals involved.

(i) The unit-step function: $x[n] = u[n]$ where $u[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$.

(ii) A four-point discrete-time box function:

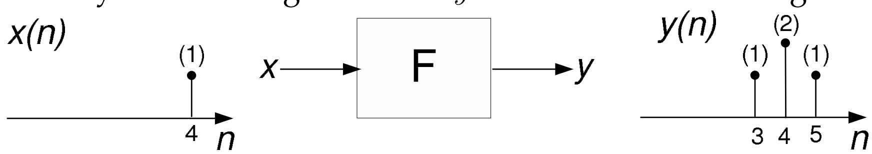
$$x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3].$$

(iii) A constant function: $x[n] = 1$.

(iv) The sign-alternating signal $x[n] = 2(-1)^n$.

(b) Based on the results of part (a), explain why the system F may aptly be called a one-dimensional *edge detector*. Does the system's response provide a clue as to which direction (up or down) each edge in the input signal is?

HW2.4 Consider a discrete-time system $\mathsf{F} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$. It is known that F is linear and time invariant. The following input-output signal pair (x, y) is a behavior of the system. The signals x and y are zero outside the regions shown.



Provide succinct, but clear and convincing responses to the questions below.

(a) Can the impulse response f be determined from the information given in this problem? If so, provide a well-labeled plot of the sample values $f[n]$. If not, explain why it is not possible to determine f based on what is known about the system F.

(b) Determine, and provide well-labeled, hand-drawn sketches of, the response of the system to each of the following input signals:

(i) $x[n] = 1$ for all $n \in \mathbb{Z}$.

(ii) $x[n] = \cos(\pi n)$ for all $n \in \mathbb{Z}$.

HW2.5 (Discrete convolution practice) Consider the signal

$$x[n] = \alpha^n u[n].$$

(a) Sketch the signal $g[n] = x[n] - \alpha x[n-1]$. Write $g[n]$ as a convolution related to $x[n]$.

(b) Use the result of part (a) along with the properties of convolution to determine $h[n]$ such that

$$x[n] * h[n] = (\frac{1}{2})^n (u[n+2] - u[n-2]).$$

Hint: Use the fact that $x[n] * \delta[n] = x[n]$.

HW2.6 (Polynomial Multiplication and Discrete-Time Convolution) The impulse response of a real[1], discrete-time FIR filter A is described by

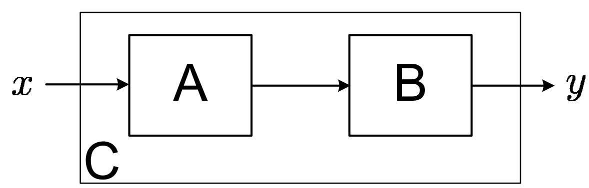
$$a[n] = a_0\delta[n] + a_1\delta[n-1] + \cdots + a_N\delta[n-N], \quad \text{where } N \in \{1, 2, 3, \ldots\}.$$

(a) Show that the frequency response $A(\omega)$ of the filter is a polynomial, in terms of $e^{-i\omega}$, whose coefficients have a simple relationship with the impulse response values $a_0, \ldots, a_N$.

(b) Suppose the filter A is placed in a cascade (series) interconnection with another discrete-time FIR filter B whose impulse response is described by

$$b[n] = b_0\delta[n] + b_1\delta[n-1] + \cdots + b_M\delta[n-M], \quad \text{where } M \in \{1, 2, 3, \ldots\}.$$

The cascade structure, which we call C, is shown in the figure below.



Let c denote the impulse response of the cascade interconnection.

(i) Find an expression for $c[n]$ in terms of $a_0, a_1, ..., a_N$ and $b_0, b_1, ..., b_M$.

[1]A real filter is one that produces a real-valued output signal for every real-valued input signal. An LTI filter is real if its impulse-response is real-valued.

(ii) Consider two polynomials $A(z)$ and $B(z)$ described as follows:

$$A(z) = a_0 + a_1 z + \cdots + a_N z^N \qquad\qquad B(z) = b_0 + b_1 z + \cdots + b_M z^M\,,$$

where M and N and positive integers. Let $C(z) = A(z)\,B(z)$, where

$$C(z) = c_0 + c_1 z + \cdots + c_{N+M} z^{N+M}.$$

Show that multiplying polynomials is tantamount to convolving their coefficients; in particular, explain how $c_n = (a * b)_n = \sum_m a_m b_{n-m}$, where $n = 0, 1, \ldots, N + M$.