Your LAST Name _____ **Your FIRST Name** _____

Your Collaborators

- Name: _____ SID: _____
- Name: _____ SID: _____
- Name: _____ SID: _____
- Name: _____ SID: _____

Policy Statement

- We encourage you to collaborate, but only in a group of up to *five* current EE 120 students.
- Be sure to write the names of your collaborators at the appropriate location on the next page. Each collaborator must submit a distinct, self-prepared solution document containing original contributions to the collaborative effort.
- To submit your work, you have several options:
  (a) Print out this homework document and write your solutions on the printout. Then upload a *PDF file* of the scan.
  (b) Write your solutions on this PDF file using a tablet device. Then upload a *PDF file*.
  (c) Use your own sheets of paper, but demarcate clearly the space for each problem, part, and subpart. Then upload a *PDF file*.
- Please write neatly and legibly, because *if we can't read it, we can't evaluate it.* Remember that, even though the problem sets are self-graded, we will evaluate your work whenever we check your self grades.
- Unless we explicitly state otherwise, you may expect full credit *only if* you explain your work succinctly, but clearly and convincingly.
- If you are asked to provide a "sketch," it refers to a well-labeled *hand-drawn* sketch—not a plot generated by a computing device.
- On occasion, a problem set contains one or more problems designated as "optional." Your solutions to such problems will NOT be evaluated. Nevertheless, you're responsible for learning the subject matter within their scope.
- If you have a confirmed disability that precludes you from complying fully with these instructions or with any other parameter associated with this problem set, please alert us immediately about reasonable accommodations afforded to you by the DSP Office on campus.

HW3.1 Periodic Periodic Periodic For each signal below, determine if it is periodic or not. If it is, determine the fundamental period. For CT signals, we define $x(t)$, $t \in \mathbb{R}$, and for DT signals, we define $x[n]$, $n \in \mathbb{Z}$.

(a) $x(t) = e^{\frac{i\pi t}{2}}$.

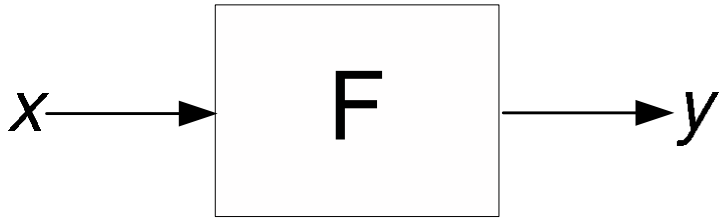
(b) $x[n] = \cos(n)$.

(c) $x[n] = \cos(\pi n)$.

(d) $x(t) = \sin(2\pi t) + \sin(3\pi t)$.

(e) **(Challenge)** $x[n] = \cos\left(\frac{\pi}{2}n^2\right)$.

HW3.2 (Frequency Response of a Discrete-Time IIR Filter) The causal, discrete-time LTI filter F shown below



is characterized by the recursive difference equation $y[n] = \alpha\, y[n-N] + x[n]$, where $|\alpha| < 1$ and $N \in \{1, 2, 3, \ldots\}$.

(a) Determine a simple expression for $f[n]$, where f is the impulse response of the filter. Express your answer in terms of α and N.
Hint: Because the system is causal, you know $f[n] = 0$ for $n < 0$. You don't need to derive the expression rigorously. Instead, evaluate a few cases for different values of N and look for a pattern.

(b) Determine a reasonably simple expression for $F(\omega)$, where F is the frequency response of the filter. Express your answer in terms of α and N.

(c) Provide well-labeled sketches of $f[n]$ for

(i) $\alpha = \frac{1}{2}$ and $N = 2$ (ii) $\alpha = \frac{1}{\sqrt{2}}$ and $N = 2$ (iii) $\alpha = \frac{1}{2}$ and $N = 4$.

Describe, in simple terms, the respective effects of α and N on the qualitative features of the impulse response f.

(d) Provide a well-labeled sketch of the magnitude response $|F(\omega)|$ for each of the following cases:

(i) $\alpha = \frac{1}{2}$ and $N = 2$ (ii) $\alpha = \frac{1}{\sqrt{2}}$ and $N = 2$ (iii) $\alpha = \frac{1}{2}$ and $N = 4$.

HW3.3 From LCCDE to Frequency Response and back! This problem will explore the connection between LCCDEs and the frequency response. The LCCDE for system F is:

$$3y[n] - 4y[n-1] - 5y[n-2] = 6x[n] + 7x[n-1] \quad \forall n \in \mathbb{Z}$$

(a) Using the eigenfunction property, find the frequency response $F(\omega)$.

(b) Now given a general LCCDE in this form:

$$a_0 y[n] + a_1 y[n-1] + \cdots + a_k y[n-k] = b_0 x[n] + b_1 x[n-1] + \cdots + b_l x[n-l]$$

Find the frequency response $F(\omega)$.

(c) Based on your results from above, find the LCCDE given the frequency response for a new LTI system H.

$$H(\omega) = \frac{3 + 8e^{3i\omega} - e^{i5\omega}}{1 + e^{i\omega}}$$