Problem Set 4
Due: Fri, 18 Sep 2026, 23:59
Accepted Thru: Sun, 20 Sep 2026, 23:59

Your LAST Name ____________________ **Your FIRST Name** ____________________

Your Collaborators

- Name: ____________________________________ SID: ____________________________
- Name: ____________________________________ SID: ____________________________
- Name: ____________________________________ SID: ____________________________
- Name: ____________________________________ SID: ____________________________

Policy Statement

- We encourage you to collaborate, but only in a group of up to *five* current EE 120 students.
- Be sure to write the names of your collaborators at the appropriate location on the next page. Each collaborator must submit a distinct, self-prepared solution document containing original contributions to the collaborative effort.
- To submit your work, you have several options:
  (a) Print out this homework document and write your solutions on the printout. Then upload a *PDF file* of the scan.
  (b) Write your solutions on this PDF file using a tablet device. Then upload a *PDF file*.
  (c) Use your own sheets of paper, but demarcate clearly the space for each problem, part, and subpart. Then upload a *PDF file*.
- Please write neatly and legibly, because *if we can't read it, we can't evaluate it.* Remember that, even though the problem sets are self-graded, we will evaluate your work whenever we check your self grades.
- Unless we explicitly state otherwise, you may expect full credit *only if* you explain your work succinctly, but clearly and convincingly.
- If you are asked to provide a "sketch," it refers to a well-labeled *hand-drawn* sketch—not a plot generated by a computing device.
- On occasion, a problem set contains one or more problems designated as "optional." Your solutions to such problems will NOT be evaluated. Nevertheless, you're responsible for learning the subject matter within their scope.
- If you have a confirmed disability that precludes you from complying fully with these instructions or with any other parameter associated with this problem set, please alert us immediately about reasonable accommodations afforded to you by the DSP Office on campus.

Overview

This problem set explores

- Periodicity
- Casuality and BIBO stability
- LCCDE / DAG representations of systems

HW4.1 (Discrete-Time Periodic Signals) Let $x[n]$ be a discrete-time signal, and let

$$y_1[n] = x[3n] \text{ and } y_2[n] = \begin{cases} x[n/3] & \text{if } n \bmod 3 = 0 \\ 0 & \text{if } n \bmod 3 \neq 0 \end{cases}$$

The signals $y_1[n]$ and $y_2[n]$ respectively represent in upsampled (sped up) and downsampled (slowed down) versions of $x[n]$. For each of the following statements, determine whether it is true. If so, provide a proof. If not, provide a counterexample.

(1) If $x[n]$ is periodic, then $y_1[n]$ is periodic.

(2) If $y_1[n]$ is periodic, then $x[n]$ is periodic.

(3) If $x[n]$ is periodic, then $y_2[n]$ is periodic.

(4) If $y_2[n]$ is periodic, then $x[n]$ is periodic.

HW4.2 (System Properties) Consider each of the following systems independently. For each system, x denotes the input, and y the corresponding output.[1]

$$\begin{aligned}
\mathsf{F}_1 &: [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]; \forall n \in \mathbb{Z}, y[n] = x[-n]. \\
\mathsf{F}_2 &: [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]; \forall n \in \mathbb{Z}, y[n] = x[2n]. \\
\mathsf{F}_3 &: [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]; \forall n \in \mathbb{Z}, y[n] = \begin{cases} 0 & \text{if } n \text{ is an odd integer} \\ 1 & \text{if } n \text{ is an even integer.} \end{cases} \\
\mathsf{F}_4 &: [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]; \forall n \in \mathbb{Z}, y[n] = \begin{cases} x\left[\frac{n}{N}\right] & n \,\mathsf{mod}\, N = 0 \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}$$

The system F_{10} is called an N-fold upsampler.

Provide a brief but convincing responses to the questions below.

(a) For each system, select the strongest true assertion from the list below.

(i) The system must be causal.

(ii) The system could be causal, but does not have to be.

(iii) The system cannot be causal.

(b) For each system, select the strongest true assertion from the list below.

(i) The system must be BIBO stable.

(ii) The system could be BIBO stable, but does not have to be.

(iii) The system cannot be BIBO stable.

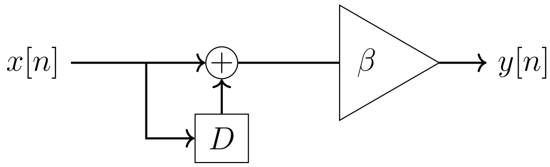
[1]The definition of system F_{10} uses the mod function. For any $n \in \mathbb{Z}$ and $N \in \mathbb{N}$, the expression "n mod N"—read "n modulo N"—is the unique integer remainder k ($0 \leq k \leq N-1$) when n is divided by N, e.g., $5 \,\mathsf{mod}\, 3 = 2$ and $-4 \,\mathsf{mod}\, 3 = 2$. Note that N divides $n - k$.

(a) (Causality)

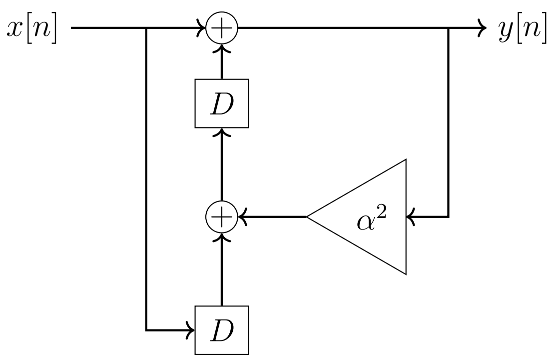
(b) (BIBO Stability)

HW4.3 For each of the following delay-adder-gain (DAG) diagrams, give the linear, constant coefficient difference equation (LCCDE) which describes the system.

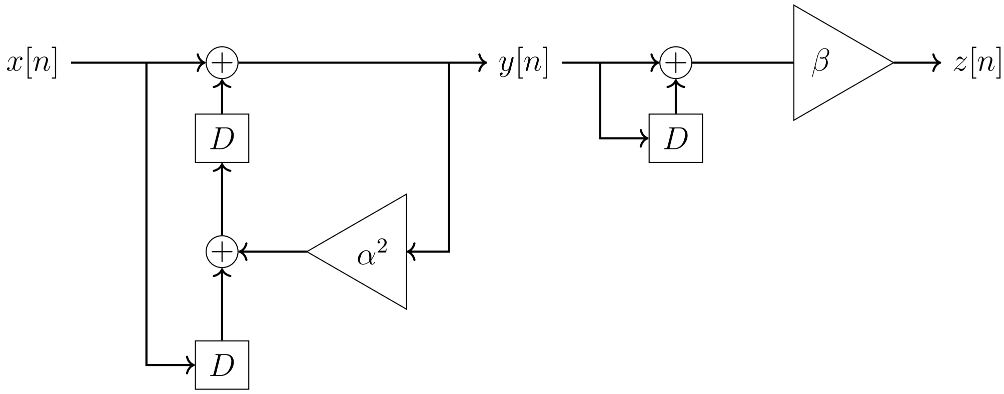
(a)



(b)



(c)



Note that this is the system from (a) composed with the system from (b). Give an equation in terms of x and z only.

HW4.4 Consider a discrete-time LTI system $\mathsf{F} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$ whose input and output signals x and y, respectively, satisfy the following linear, constant-coefficient difference equation:

$$y[n] + \alpha^2\, y[n-2] = x[n] + x[n-2]\,,$$

where $0 < \alpha < 1$.

(a) Draw a delay-adder-gain (DAG) block diagram implementation of the system.

(b) Determine an expression for the frequency response $F : \mathbb{R} \to \mathbb{C}$ of the system.

(c) Suppose $\alpha = 0.95$. Provide a well-labeled sketch of $|F(\omega)|$, the magnitude of the frequency response of the system. Explain why this filter is called a *notch filter*.
Hint: consider what happens at $\omega = \pm\frac{\pi}{2}$ versus elsewhere. Think of $e^{i2\omega}$ as a unit vector rotating in the complex plane, and visualize both the numerator and denominator as vectors. Try sketching this visualization for different values of α.

(d) Suppose $\alpha = 0.95$ and that the input signal is characterized by

$$\forall n \in \mathbb{Z}, \quad x[n] = \cos\left(\frac{\pi}{2}n\right) + \frac{1}{3}\sin\left(\frac{\pi}{4}n\right) + 3 + (-1)^n \,.$$

Using little to no mathematical manipulation, determine a reasonable approximation for the corresponding output signal values $y[n]$.

What assumption did you have to make about the phase response $\angle F$ that enabled you to approximate the output signal y? Why was your assumption about the phase reasonable?